

## QUANTUM MECHANICS AND THE INTUITION AND METHOD OF EINSTEIN

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Late in his life, with characteristic humility, Einstein claimed but a single virtue as a physicist: he said he was stubborn as a mule. As an after thought, he added, "I also have a keen scent."<sup>i</sup> Few would dispute that Einstein's scent—his intuition—had few equals in the history of science. The intent here is to examine a single aspect of Einstein's intuition and gauge to what extent it may still be useful to us. The intuition in question centers on Einstein's pursuit of the symmetrical relationship between the molecular gas and the photon gas.

Einstein's earliest interests in physics centered on thermodynamics, statistical mechanics, and kinetic theory. His creative research on these subjects spanned a quarter century and some 40 articles, and all of his principal contributions to quantum theory were statistical in origin.<sup>ii</sup> In his famous paper of March 1905, Einstein introduced and applied (to the photoelectric effect) the hypothesis that light consists of discrete energy quanta. Einstein based his light quantum theory on arguments drawn from statistical mechanics, arguments whose novelty is reflected in the title of the paper, "On a Heuristic Viewpoint Concerning the Production and Transformation of Light." Commentators often characterize the arguments of the paper as creating an analogy between radiation and a classical ideal gas of material particles. But to Einstein, the connection was more than an analogy; it was a "far-reaching formal relationship."<sup>iii</sup>

Einstein began with a derivation of the entropy decrease when  $N$  discrete molecules of an ideal gas contract from a larger volume  $\nu_1$  to a smaller volume,  $\nu_2$ . The probability expression for this contraction is  $W = \left(\frac{\nu_2}{\nu_1}\right)^N$ . Using Wein's radiation function relating energy density, temperature,

and frequency, Einstein derived an expression for the entropy decrease when monochromatic radiation of energy  $E(q)$  in volume  $\nu_1$  is squeezed into a smaller volume  $\nu_2$ . He showed the

probability expression for this latter process is  $W = \left(\frac{\nu_2}{\nu_1}\right)^{\frac{E(q)}{hq}}$ . Einstein then proposed that the

parallelism of mathematical form and thermodynamic behavior between the two cases suggest that radiation is identical in form to an ideal gas, and that it is composed of discrete particles, namely, energy quanta. Specifically, the number of radiation energy quanta  $N$  must be equal to  $E(q)/hq$ , hence,  $E(q) = Nhq$ . A system of  $N$  quantum units then has  $hq$  energy units associated with each "particle." This is a brilliant, inspired hypothesis that had to wait more than a decade for experimental confirmation.

In 1909, Einstein published two papers expanding on his view that light is composed of energy quanta. One of his arguments again drew on the thermodynamic relations connecting an ideal gas with radiation, this time from a black body. He imagined a flat plate as a perfect reflector buffeted simultaneously by the collisions from gas molecules and by the pressure from blackbody radiation. The argument joined together his earlier work on molecular Brownian motion with his central concept of light quanta as a type of radiation gas.

Two more papers followed in 1916-17, again focusing on blackbody radiation. Einstein began with a reference to the ideal gas, noting the formal resemblance between the frequency distribution of blackbody radiation and Maxwell's distribution law for the velocities of gas molecules. This observation was by way of justifying one of his favorite procedures, namely, considering a molecular gas in equilibrium with blackbody radiation. Such a thought experiment had molecules both emitting and absorbing quantized radiation energy while still maintaining energy equilibrium. Combining these circumstances, a few generalized assumptions about emission and absorption, and the statistical interpretation of the second law of thermodynamics, Einstein brought forth a new derivation of Planck's law. He then addressed the problem of how a Maxwell distribution of molecular velocity could maintain itself subject to the effects of radiation pressure. In other words, how would an ideal gas and a photon gas interact in the equilibrium state in reference to molecular velocity distribution? The answer led Einstein to postulate that light quanta carry a

momentum of  $h\nu/c$ , a hypothesis anticipated by Stark (1909) and subsequently confirmed by Compton in 1923. As in 1905 and 1909, Einstein's papers in 1916-17 on radiation quanta again depended upon the formal relationships between the molecular and the radiation gases.

The 1917 paper was Einstein's last on radiation, although he never ceased to ponder the mysteries of light quanta.<sup>iv</sup> By the early 1920s, he was deeply involved in his unified field theory, and he was not a direct participant in quantum theory research. But in the summer of 1924, Einstein received from an Indian physicist, S. N. Bose, a manuscript whose publication Einstein arranged. In this paper, Bose elaborated a new technique of counting the statistical distribution of total energy  $E$  over  $N$  particles (light quanta). The result constitutes "a natural development of the ideas that Einstein had been advocating for close to twenty years..."<sup>v</sup> For a few months, Einstein put aside his work on the unified field and turned back to statistical mechanics. But whereas Bose used his new counting procedure merely to derive Planck's law, Einstein used the procedure to deepen the relationship he knew existed between molecular and photon gases.

In his three papers of 1924-25, Einstein treated the molecular gas, not the photon gas, as he moved into the uncharted waters of quantum statistics. He analyzed and explained the low temperature gas phenomenon now known as the Bose-Einstein condensation. This analysis was immensely satisfying to him on the grounds of symmetry: a Bose-Einstein molecular gas yielded the third law of thermodynamics, just as a Bose-Einstein photon gas yielded Planck's law. But the most important achievement of Einstein's last creative hurrah led him close to a topic about which he had never felt entirely comfortable, wave mechanics.

When exploring the energy fluctuations of the photon gas in 1909, Einstein discovered an expression with two terms, one dependent upon radiation's continuous (wave interference) nature, and the other—surprisingly—dependent upon radiation's presumed discrete nature. In 1924, his conjugal analysis examined the density fluctuations of the molecular gas. This time, he found the expected term deriving from discrete effects; but in addition, there appeared a term suggesting continuous/wave effects. Having read de Broglie's thesis some months earlier, Einstein did not hesitate in associating a wave field with the rapidly moving gas molecules. Einstein's original ideas of fluctuations and duality had now come full circle. The energy fluctuations in the photon gas were

mirrored in the density fluctuations of the molecular gas; and the particle/wave character of photons had its counterpart in the wave/particle nature of gas molecules.

Einstein's acceptance of de Broglie's wave/particle duality for matter, and his use of it in his quantum theory of the ideal gas, broke the logjam that had been impeding quantum theory development. For some years, Schrödinger, too, had attempted to apply quantum theory to the ideal gas. He read Einstein's papers and saw a way to use de Broglie ideas to evade Bose's strange counting procedure.

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Spanning two decades of research and publishing, Einstein's central idea was the relationship of the photon gas to the molecular/ideal gas. He based his ideas on considerations of symmetry, but he knew that symmetry alone would not convince any of his readers. His genius lay in his ability to derive physical laws and testable predictions from the symmetrical thermodynamic relationships of photon gas and molecular gas. Einstein recognized the differences of the two gases in content but believed they shared the same formal principles. Einstein drew his method from his intuition: the search for entities and processes different in content but identical in form. It was a most powerful tool in the hands of a master; and while it has passed into neglect since 1925, there are no signs its efficacy has been exhausted.

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<sup>i</sup> E.G. Straus in *Helle Zeit-Dunkle Zeit*, ed. C. Seelig (Zurich: Europa Verlag, 1956), p. 72.

<sup>ii</sup> Abraham Pais, 'Subtle is the Lord...' *The Science and the Life of Albert Einstein*, (New York, 1982), p. 56.

<sup>iii</sup> A. Einstein, *Sitz. Ber. Preuss. Ak. Wiss.* 1925, p. 3.

<sup>iv</sup> In a letter to Besso in 1951, Einstein wrote: "All these fifty years of pondering have not brought me any closer to answering the question, 'What are light quanta?'"

<sup>v</sup> Martin J. Klein, "Einstein and the Wave-Particle Duality", in *Natural Philosopher*, v.3, 1964, p.27.